Complete Solutions to Supplementary Exercises on Differentiation

1. (a) We need to find $\frac{dy}{dx}$ given that $y=\left(1+\sqrt[3]{x}\right)^3$. Rewriting y we have

$$y=\left(1+\sqrt[3]{x}\right)^3=\left(1+x^{\frac{1}{3}}\right)^3$$

Differentiating this gives

$$\begin{aligned}\frac{dy}{dx}&=\cancel{3}\left(1+x^{\frac{1}{3}}\right)^2\frac{1}{\cancel{3}}x^{-\frac{2}{3}}\\&=\frac{1+2x^{\frac{1}{3}}+x^{\frac{2}{3}}}{x^{\frac{2}{3}}}=\frac{1}{x^{\frac{2}{3}}}+\frac{2x^{\frac{1}{3}}}{x^{\frac{2}{3}}}+\frac{\cancel{x^{\frac{2}{3}}}}{\cancel{x^{\frac{2}{3}}}}=1+\frac{2}{\sqrt[3]{x}}+\frac{1}{\left(\sqrt[3]{x}\right)^2}\end{aligned}$$

(b) We are asked to differentiate $y=a\tan\left(\frac{x}{k}+b\right)$:

$$\frac{dy}{dx}=a\sec^2\left(\frac{x}{k}+b\right)\left(\frac{1}{k}\right)=\frac{a}{k}\sec^2\left(\frac{x}{k}+b\right)=\frac{a}{k}\left[1+\tan^2\left(\frac{x}{k}+b\right)\right]$$

(c) We need to differentiate $y=\log_{10}\left(x-\cos\left(x\right)\right)$. First converting the log of base 10 to natural logarithm:

$$y=\log_{10}\left(x-\cos\left(x\right)\right)=\frac{\ln\left(x-\cos\left(x\right)\right)}{\ln\left(10\right)}$$

Differentiating this by taking out $\frac{1}{\ln\left(10\right)}$ we have

$$\frac{dy}{dx}=\frac{1}{\ln\left(10\right)}\left[\frac{1+\sin\left(x\right)}{x-\cos\left(x\right)}\right]$$

(d) For $y=\sin\left(x\right).e^{\cos\left(x\right)}$ we use the product rule with

$$\begin{aligned}u&=\sin\left(x\right) \qquad & v&=e^{\cos\left(x\right)}\\u'&=\cos\left(x\right) \qquad & v'&=-e^{\cos\left(x\right)}\sin\left(x\right)\end{aligned}$$

Therefore

$$\begin{aligned}\frac{dy}{dx}=u'v+uv'&=\cos\left(x\right)e^{\cos\left(x\right)}+\sin\left(x\right)\left[-e^{\cos\left(x\right)}\sin\left(x\right)\right]\\&=e^{\cos\left(x\right)}\left[\cos\left(x\right)-\sin^2\left(x\right)\right]\\&=e^{\cos\left(x\right)}\left[\cos^2\left(x\right)+\cos\left(x\right)-1\right] \qquad \left[\text{Using }\sin^2\left(x\right)=1-\cos^2\left(x\right)\right]\end{aligned}$$

(e) We are asked to differentiate $y = e^{-x^2} \ln\left(x\right)$. Again using the product rule with:

$$u = e^{-x^2} \qquad v = \ln\left(x\right)$$
$$u' = -2xe^{-x^2} \qquad v' = \frac{1}{x}$$

Applying the product rule we have

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= -2xe^{-x^2}\ln\left(x\right) + e^{-x^2}\frac{1}{x} \\ &= e^{-x^2}\left[\frac{1}{x} - 2x\ln\left(x\right)\right] = \frac{e^{-x^2}}{x}\left[1 - 2x^2\ln\left(x\right)\right]\end{aligned}$$

(f) We need to differentiate $y = x\tan^{-1}\left(\sqrt{x}\right)$. Applying the product rule with:

$$u = x \qquad v = \tan^{-1}\left(\sqrt{x}\right)$$

The derivative of u is straightforward but the derivative of v is as follows:

$$\tan\left(v\right) = \sqrt{x} = x^{\frac{1}{2}}$$
$$\sec^2\left(v\right)\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{2\sqrt{x}} \quad \Rightarrow \quad \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{2\sqrt{x}\sec^2\left(v\right)} = \frac{1}{2\sqrt{x}\left[1 + \tan^2\left(v\right)\right]}$$

Recall $\tan\left(v\right) = \sqrt{x}$ so $v = \tan^{-1}\left(\sqrt{x}\right)$. Substituting this $v = \tan^{-1}\left(\sqrt{x}\right)$ into the above gives

$$\begin{aligned}v' = \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{2\sqrt{x}\left[1 + \tan^2\left(v\right)\right]} &= \frac{1}{2\sqrt{x}\left[1 + \tan^2\left(\tan^{-1}\left(\sqrt{x}\right)\right)\right]} \\ &= \frac{1}{2\sqrt{x}\left[1 + x\right]} \qquad \left[\text{Because } \tan\left(\tan^{-1}\left(\theta\right)\right) = \theta\right]\end{aligned}$$

Now using the product rule we have

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = u'v + uv' &= \tan^{-1}\left(\sqrt{x}\right) + x\frac{1}{2\sqrt{x}\left[1 + x\right]} \\ &= \tan^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{x}}{2\left[1 + x\right]}\end{aligned}$$

(g) We use the chain rule to find the derivative of $y = \sqrt[3]{1 + x\sqrt{x + 3}}$. Firstly rewriting this we have

$$y = \sqrt[3]{1 + x\sqrt{x + 3}} = \left(1 + x\left(x + 3\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}$$

Then

$$\frac{dy}{dx} = \frac{1}{3}\left(1 + x\left(x+3\right)^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(1 + x\left(x+3\right)^{\frac{1}{2}}\right)'$$

$$= \frac{1}{3}\left(1 + x\left(x+3\right)^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(1 + \left(x^3+3x^2\right)^{\frac{1}{2}}\right)' \qquad \left[\begin{array}{l}\text{Taking the } x \text{ under the}\\ \text{square root in the last bracket}\end{array}\right]$$

$$= \frac{1}{3}\left(1 + x\left(x+3\right)^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(0 + \frac{1}{2}\left(x^3+3x^2\right)^{-\frac{1}{2}}\left[3x^2+6x\right]\right)$$

$$= \frac{3x^2+6x}{6\left(1 + x\left(x+3\right)^{\frac{1}{2}}\right)^{\frac{2}{3}}\sqrt{\left(x^3+3x^2\right)}}$$

(h) We need to find derivative of $y = \sin^{-1}\left(x\right) + \sqrt{1-x^2}$. Differentiating each one separately. Let

$$u = \sin^{-1}\left(x\right) \quad \Rightarrow \quad \sin\left(u\right) = x$$

$$\cos\left(u\right)\frac{du}{dx} = 1 \quad \Rightarrow \quad \frac{du}{dx} = \frac{1}{\cos\left(u\right)}$$

Recall that $\cos\left(u\right) = \sqrt{1-\sin^2\left(u\right)}$. Substituting this into the above yields

$$\frac{du}{dx} = \frac{1}{\cos\left(u\right)} = \frac{1}{\sqrt{1-\sin^2\left(u\right)}}$$

$$= \frac{1}{\sqrt{1-\sin^2\left(\sin^{-1}\left(x\right)\right)}} \qquad \left[\text{Because } u = \sin^{-1}\left(x\right)\right]$$

$$= \frac{1}{\sqrt{1-x^2}} \qquad \left[\text{Because } \sin\left(\sin^{-1}\left(\theta\right)\right) = \theta\right]$$

Now let $v = \sqrt{1-x^2} = \left(1-x^2\right)^{\frac{1}{2}}$ and differentiating this gives

$$\frac{dv}{dx} = \frac{-2x}{2\left(1-x^2\right)^{\frac{1}{2}}} = -\frac{x}{\sqrt{1-x^2}}$$

Adding both of these derivatives $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ and $\frac{dv}{dx} = -\frac{x}{\sqrt{1-x^2}}$ together

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$
$$= \frac{1-x}{\sqrt{1-x^2}}$$
$$= \frac{1-x}{\sqrt{1-x}\sqrt{1+x}} = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \sqrt{\frac{1-x}{1+x}}$$

(i) We need to apply logarithmic differentiation to $y = 2^{\frac{x}{\ln(x)}}$. Applying logs to both sides gives

$$\ln(y) = \ln\left(2^{\frac{x}{\ln(x)}}\right) = \frac{x}{\ln(x)}\ln(2)$$

Differentiating this yields

$$\frac{1}{y}\frac{dy}{dx} = \ln(2)\underbrace{\left[\frac{\ln(x) - \frac{\cancel{x}}{\cancel{x}}}{\ln^2(x)}\right]}_{\text{Quotient rule}} = \frac{\ln(2)}{\ln^2(x)}\left[\ln(x) - 1\right]$$

Therefore we have

$$\frac{dy}{dx} = y\frac{\ln(2)}{\ln^2(x)}\left[\ln(x) - 1\right] = 2^{\frac{x}{\ln(x)}}\frac{\ln(2)}{\ln^2(x)}\left[\ln(x) - 1\right]$$

(j) Again we apply logarithmic differentiation to $y = \left[\tan(2x)\right]^{\cot\left(\frac{x}{2}\right)}$ in order to find the derivative. Taking logs of both sides gives

$$\ln(y) = \ln\left(\left[\tan(2x)\right]^{\cot\left(\frac{x}{2}\right)}\right) = \cot\left(\frac{x}{2}\right)\ln\left[\tan(2x)\right]$$
$$\frac{1}{y}\frac{dy}{dx} = \left(\cot\left(\frac{x}{2}\right)\ln\left[\tan(2x)\right]\right)' \qquad (*)$$

We differentiate the right hand side by using the product rule. Let

$u = \cot\left(\frac{x}{2}\right)$ and $v = \ln\left[\tan(2x)\right]$ then

$$u' = -\frac{1}{2}\operatorname{cosec}^2\left(\frac{x}{2}\right) \quad \text{and} \quad v' = \frac{2\sec^2(2x)}{\tan(2x)}$$

Using the product rule we have

$$\left(\cot\left(\frac{x}{2}\right)\ln\left[\tan\left(2x\right)\right]\right)' = u'v + uv'$$
$$= -\frac{1}{2}\cos ec^2\left(\frac{x}{2}\right)\ln\left[\tan\left(2x\right)\right] + 2\cot\left(\frac{x}{2}\right)\frac{\sec^2\left(2x\right)}{\tan\left(2x\right)}$$

Putting this into (*) gives

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}\cos ec^2\left(\frac{x}{2}\right)\ln\left[\tan\left(2x\right)\right] + 2\cot\left(\frac{x}{2}\right)\frac{\sec^2\left(2x\right)}{\tan\left(2x\right)}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = y\left[2\cot\left(\frac{x}{2}\right)\frac{\sec^2\left(2x\right)}{\tan\left(2x\right)} - \frac{1}{2}\cos ec^2\left(\frac{x}{2}\right)\ln\left[\tan\left(2x\right)\right]\right]$$
$$= \left[\tan\left(2x\right)\right]^{\cot\left(\frac{x}{2}\right)}\left[2\cot\left(\frac{x}{2}\right)\frac{\sec^2\left(2x\right)}{\tan\left(2x\right)} - \frac{1}{2}\cos ec^2\left(\frac{x}{2}\right)\ln\left[\tan\left(2x\right)\right]\right]$$

(k) We are asked to differentiate $y = \cos^{-1}\left(\frac{x^{2n}-1}{x^{2n}+1}\right)$. Taking cos of both sides:

$$\cos\left(y\right) = \frac{x^{2n}-1}{x^{2n}+1}$$

Differentiating this gives

$$-\sin\left(y\right)\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x^{2n}-1}{x^{2n}+1}\right)' \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sin\left(y\right)}\left(\frac{x^{2n}-1}{x^{2n}+1}\right)' \qquad (\dagger)$$

We differentiate the bracketed term by using the quotient rule with $u = x^{2n}-1$ and $v = x^{2n}+1$. Therefore we have

$$u' = 2nx^{2n-1} \text{ and } v' = 2nx^{2n-1}$$

Hence

$$\left(\frac{x^{2n}-1}{x^{2n}+1}\right)' = \frac{u'v - uv'}{v^2}$$
$$= \frac{2nx^{2n-1}\left(x^{2n}+1\right) - \left(x^{2n}-1\right)2nx^{2n-1}}{\left(x^{2n}+1\right)^2}$$
$$= \frac{2nx^{4n-1} + 2nx^{2n-1} - 2nx^{4n-1} + 2nx^{2n-1}}{\left(x^{2n}+1\right)^2}$$
$$= \frac{4nx^{2n-1}}{\left(x^{2n}+1\right)^2}$$

Substituting this $\left(\frac{x^{2n}-1}{x^{2n}+1}\right)' = \frac{4nx^{2n-1}}{\left(x^{2n}+1\right)^2}$ into $(\dagger)$ gives

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sin\left(y\right)}\frac{4nx^{2n-1}}{\left(x^{2n}+1\right)^2} \qquad (\dagger\dagger)$$

We are given that $y = \cos^{-1}\left(\frac{x^{2n}-1}{x^{2n}+1}\right)$ so writing $\sin\left(y\right)$ in terms of $\cos\left(y\right)$ by using the fundamental trigonometric identity:

$$\sin\left(y\right) = \sqrt{1-\cos^2\left(y\right)}$$

Therefore

$$\sin\left(y\right) = \sqrt{1-\cos^2\left(y\right)} = \sqrt{1-\left(\frac{x^{2n}-1}{x^{2n}+1}\right)^2} \qquad \left[\text{Because } \; y = \cos^{-1}\left(\frac{x^{2n}-1}{x^{2n}+1}\right)\right]$$

$$= \sqrt{\frac{\left(x^{2n}+1\right)^2}{\left(x^{2n}+1\right)^2} - \frac{\left(x^{2n}-1\right)^2}{\left(x^{2n}+1\right)^2}} \qquad \left[\text{Because } 1 = \frac{\left(x^{2n}+1\right)^2}{\left(x^{2n}+1\right)^2}\right]$$

$$= \sqrt{\frac{x^{4n}+2x^{2n}+1-\left(x^{4n}-2x^{2n}+1\right)}{\left(x^{2n}+1\right)^2}}$$

$$= \sqrt{\frac{4x^{2n}}{\left(x^{2n}+1\right)^2}} = \sqrt{\frac{\left(2x^n\right)^2}{\left(x^{2n}+1\right)^2}} = \frac{2x^n}{x^{2n}+1}$$

Putting this $\sin\left(y\right) = \frac{2x^n}{x^{2n}+1}$ into $(\dagger\dagger)$ gives

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\frac{2x^n}{\cancel{x^{2n}+1}}}\frac{4nx^{2n-1}}{\left(x^{2n}+1\right)^{\cancel{2}}} = -\frac{2nx^{n-1}}{x^{2n}+1}$$

(l) We are asked to differentiate $y = \tan^{-1}\left(\tanh\left(x\right)\right)$. Taking tan of both sides

$$\tan\left(y\right) = \tanh\left(x\right)$$

Differentiating this gives

$$\sec^2\left(y\right)\frac{\mathrm{d}y}{\mathrm{d}x} = \sec h^2\left(x\right) \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sec h^2\left(x\right)}{\sec^2\left(y\right)}$$

From the trigonometric identity

(4.65) $$1+\tan^2\left(x\right) = \sec^2\left(x\right)$$

We have

$$\frac{dy}{dx} = \frac{\sec h^2(x)}{\sec^2(y)}$$

$$= \frac{\sec h^2(x)}{1+\tan^2(y)} = \frac{\sec h^2(x)}{1+\tanh^2(x)} \qquad \left[\text{Because } \tan(y) = \tanh(x)\right]$$

We have the hyperbolic identity

$$\tanh^2(x) + \sec h^2(x) = 1$$

Rearranging this yields

$$\sec h^2(x) = 1 - \tanh^2(x)$$

Putting this into the above

$$\frac{dy}{dx} = \frac{\sec h^2(x)}{1+\tanh^2(y)} = \frac{1-\tanh^2(x)}{1+\tanh^2(x)}$$

(m) We are asked to differentiate $y = \cosh\left(\sinh(x)\right)$. Using

$$\left[\cosh(u)\right]' = \sinh(u)\frac{du}{dx}$$

We have

$$\frac{dy}{dx} = \sinh\left(\sinh(x)\right)\cosh(x)$$

(n) We need to differentiate $y = \sqrt[4]{\left(1+\tanh^2(x)\right)^3}$. Rewriting this

$$y = \sqrt[4]{\left(1+\tanh^2(x)\right)^3} = \left(1+\tanh^2(x)\right)^{\frac{3}{4}}$$

Differentiating this

$$\frac{dy}{dx} = \frac{3}{4}\left(1+\tanh^2(x)\right)^{-\frac{1}{4}}\left[2\tanh(x)\sec h^2(x)\right]$$

$$= \frac{3}{4\left(1+\tanh^2(x)\right)^{\frac{1}{4}}} 2\tanh(x)\left(1-\tanh^2(x)\right)$$

$$= \frac{3\tanh(x)\left(1-\tanh^2(x)\right)}{2\left(1+\tanh^2(x)\right)^{\frac{1}{4}}}$$

2. (a) We are given $y = \ln\left(\frac{1}{1+x}\right)$ and need to show $xy' + 1 = e^y$. Differentiating y gives

$$y' = \frac{1}{1/(1+x)}\left[(1+x)^{-1}\right]'$$
$$= (1+x)\left[-\frac{1}{(1+x)^2}\right] = -\frac{1}{1+x}$$

Substituting this $y' = -\frac{1}{1+x}$ into the left hand side of the given $xy' + 1 = e^y$:

$$xy' + 1 = x\left(-\frac{1}{1+x}\right) + 1 = \frac{-x+1+x}{1+x} = \frac{1}{1+x}$$

Recall that we are given $y = \ln\left(\frac{1}{1+x}\right)$ therefore

$$e^y = e^{\ln\left(\frac{1}{1+x}\right)} = \frac{1}{1+x} \qquad \left[\text{Because } e^{\ln(u)} = u\right]$$

Hence from these two results we have $xy' + 1 = e^y$.

(b) We are given $y = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}$ and need to show $(1-x^2)y' - xy = 1$.

From the solution of question 1(h) we have

$$\left[\sin^{-1}(x)\right]' = \frac{1}{\sqrt{1-x^2}}$$

Using the quotient rule to differentiate $y = \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}$ with

$$u = \sin^{-1}(x) \qquad v = \sqrt{1-x^2}$$
$$u' = \frac{1}{\sqrt{1-x^2}} \qquad v' = -\frac{x}{\sqrt{1-x^2}}$$

Substituting these into the quotient rule formula gives

$$y' = \frac{u'v - uv'}{v^2}$$
$$= \frac{\frac{1}{\sqrt{1-x^2}}\sqrt{1-x^2} - \sin^{-1}(x)\left[-\frac{x}{\sqrt{1-x^2}}\right]}{1-x^2}$$
$$= \frac{\sqrt{1-x^2} + x\sin^{-1}(x)}{(1-x^2)^{\frac{3}{2}}}$$

Substituting this $y' = \dfrac{\sqrt{1-x^2} + x\sin^{-1}(x)}{(1-x^2)^{\frac{3}{2}}}$ and $y = \dfrac{\sin^{-1}(x)}{\sqrt{1-x^2}}$ into the left hand side of $(1-x^2)y' - xy = 1$ gives

$$(1-x^2)y' - xy = (1-x^2)\frac{\sqrt{1-x^2} + x\sin^{-1}(x)}{(1-x^2)^{\frac{3}{2}}} - x\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}$$

$$= \frac{\sqrt{1-x^2} + x\sin^{-1}(x)}{\sqrt{(1-x^2)}} - x\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \qquad \left[\begin{array}{l}\text{Using the rules}\\ \text{of indices}\end{array}\right]$$

$$= \frac{\sqrt{1-x^2}}{\sqrt{(1-x^2)}} = 1$$

Hence we have shown our result.

3. Differentiating $\dfrac{x^2}{2} + \dfrac{y^2}{4} = 1$ gives

$$2x + \frac{2y}{4}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{y/2} = -\frac{4x}{y}$$

We need to find the slope at the point $(1, \ \sqrt{2})$ which implies $x = 1$ and $y = \sqrt{2}$. Therefore

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4x}{y} = -\frac{4\times 1}{\sqrt{2}} = -\frac{2^2}{2^{\frac{1}{2}}} = -2^{\frac{3}{2}}$$

4. We are asked to find the slope of $(x-1)^2 + (y+3)^2 = 17$ at $(2, \ 1)$.

Differentiating the given function

$$2(x-1) + 2(y+3)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-x}{3+y}$$

Substituting $x = 2, \ y = 1$ into this $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1-x}{3+y}$ gives

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-x}{3+y} = \frac{1-2}{3+1} = -\frac{1}{4}$$

The slope of the circle is $-\dfrac{1}{4}$ at the point $\left(2,\ \ 1\right)$.

5. (a) First we need to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ given $x=\dfrac{1+t}{t^3},\ y=\dfrac{3}{2t^2}+\dfrac{2}{t}$:

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{t^3-3t^2\left(1+t\right)}{t^6}=\frac{t^2\left[t-3-3t\right]}{t^6}=-\frac{2t+3}{t^4}\qquad\left[\begin{array}{l}\text{Using the}\\ \text{quotient rule}\end{array}\right]$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\left(-2\right)\frac{3}{2t^3}-\frac{2}{t^2}=-\frac{1}{t^3}\left[3+2t\right]$$

Using parametric differentiation we have

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\dfrac{\mathrm{d}y}{\mathrm{d}t}}{\dfrac{\mathrm{d}x}{\mathrm{d}t}}=\frac{-\dfrac{1}{t^3}\left[3+2t\right]}{-\dfrac{2t+3}{t^4}}=t$$

Substituting this $y'=\dfrac{\mathrm{d}y}{\mathrm{d}x}=t$ and $x=\dfrac{1+t}{t^3}$ into the left hand side of $xy'^3=1+y'$:

$$xy'^3=\frac{1+t}{\cancel{t^3}}\cancel{t^3}=1+t$$

Now the right hand side $1+y'$ is equal to

$$1+y'=1+t$$

Hence we have $xy'^3=1+y'$.

(b) We are given $x=\cosh\left(2t\right),\ y=\sinh\left(2t\right)$ so

$$\frac{\mathrm{d}x}{\mathrm{d}t}=2\sinh\left(2t\right),\ \frac{\mathrm{d}y}{\mathrm{d}t}=2\cosh\left(2t\right)$$

Applying parametric differentiation we have

$$y'=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\dfrac{\mathrm{d}y}{\mathrm{d}t}}{\dfrac{\mathrm{d}x}{\mathrm{d}t}}=\frac{\cancel{2}\cosh\left(2t\right)}{\cancel{2}\sinh\left(2t\right)}=\frac{\cosh\left(2t\right)}{\sinh\left(2t\right)}$$

Therefore

$$yy'=\cancel{\sinh\left(2t\right)}\frac{\cosh\left(2t\right)}{\cancel{\sinh\left(2t\right)}}=\cosh\left(2t\right)=x$$

We have shown our required result.