Exercise 17(b)

1. (a) (i) Find the Fourier series of the following waveform which has a period of 2π:

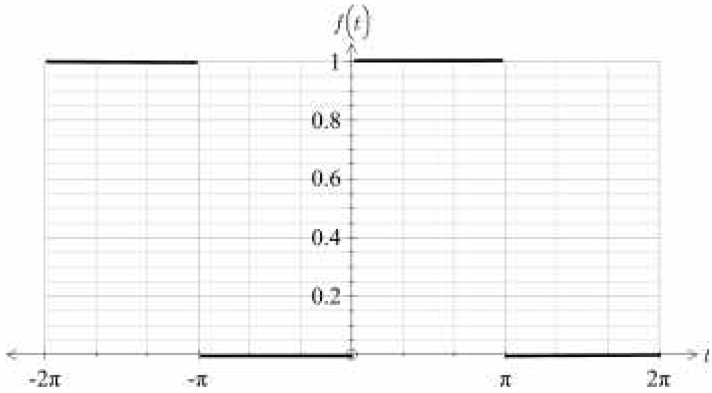


Figure 16

(ii) By substituting an appropriate value for t into your Fourier series of part (i) deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(b) (i) Determine the Fourier series of the following waveform:

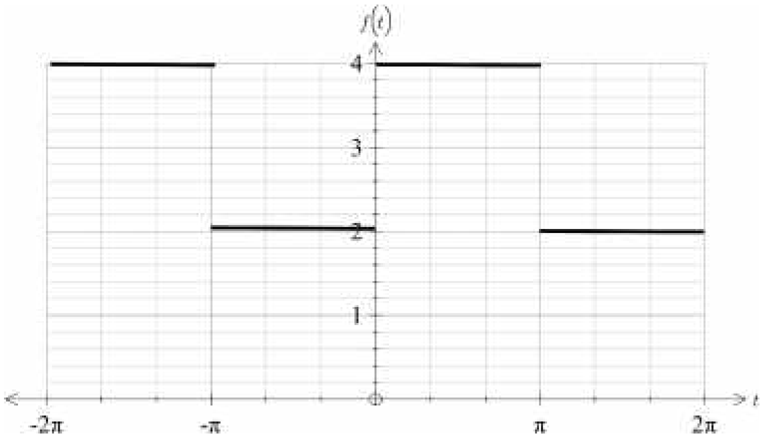
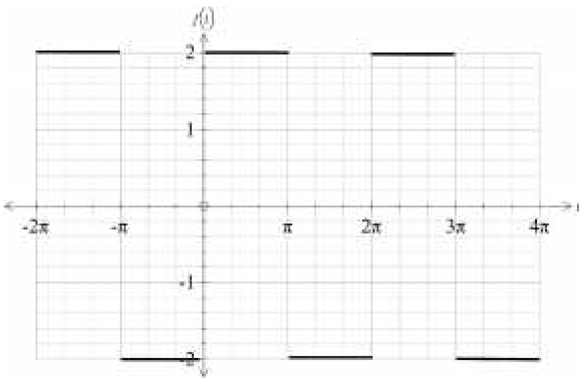


Figure 17

(ii) This time by using your Fourier series of part (b) (i) deduce

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

2. Determine the Fourier series of the following function by using the integration definition of the Fourier coefficients:



$$f(t) = \begin{cases} 2 & 0 < t < \pi \\ -2 & \pi < t < 2\pi \end{cases}$$

Figure 18

3. The function $f(t)$ is given by

$$f(t) = \begin{cases} 1 & -\pi < t < 0 \\ 2 & 0 < t < \pi \end{cases}$$

and has period 2π.

(i) Sketch $f(t)$ for $-2\pi < t < 4\pi$.

(ii) Obtain the Fourier series of $f(t)$.

4. (Mechanical). A spring-mass mechanical system is subject to an external force $F(t)$ given by

$$F(t) = \begin{cases} 20 & 0 < t < \pi \\ -20 & \pi < t < 2\pi \end{cases}$$

with period 2π.

(i) Sketch the force $F(t)$ for $-2\pi < t < 4\pi$.

(ii) Obtain the Fourier series of $F(t)$.

5. Note that the functions in questions **1, 2** and **3** are all rectangular waveforms.
In general these can be drawn as follows with amplitude A:
Figure 16
By considering this general rectangular form which has period 2π and amplitude A, show that the Fourier series of $f(t)$ is given by

$$\frac{4A}{\pi}\left[\sin(t) + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \ldots\right]$$

Hint: Use your solutions to questions **1**, **2** and **3**].

6. By using the general Fourier series of question 5 find the Fourier series of the following functions with period 2π.

(a) $f(t)=\begin{cases} 2 & 0<t<\pi \\ -2 & \pi<t<2\pi \end{cases}$ (b) $f(t)=\begin{cases} -5 & -\pi<t<0 \\ 5 & 0<t<\pi \end{cases}$

7. Show that the Fourier series of the function shown in Figure 19 which has an amplitude of A is given by

$$\frac{A}{2}+\frac{2A}{\pi}\left[\sin(t)+\frac{\sin(3t)}{3}+\frac{\sin(5t)}{5}+\ldots\right]$$

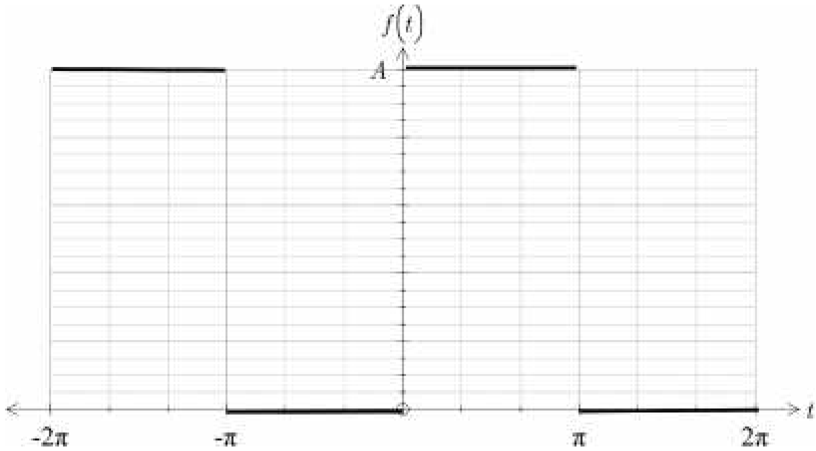


Figure 19

8. Show the following result for positive integers m and n:

$$\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,\mathrm{d}x=\begin{cases} 0 & \text{if } m\neq n \\ \pi & \text{if } m=n \end{cases}$$

9. Show the following result for positive integers m and n:

$$\int_{-\pi}^{\pi}\sin(mx)\sin(nx)\,\mathrm{d}x=\begin{cases} 0 & \text{if } m\neq n \\ \pi & \text{if } m=n \end{cases}$$

10. Show the following result for any integers m and n:

$$\int_{-\pi}^{\pi}\sin(mx)\cos(nx)\,\mathrm{d}x=0$$

Brief Solutions to Exercise 17(b)

1. (a) $\frac{1}{2}+\frac{2}{\pi}\left[\sin(t)+\frac{\sin(3t)}{3}+\frac{\sin(5t)}{5}+\ldots\right]$

(b) $3+\frac{4}{\pi}\left[\sin\left(t\right)+\frac{\sin\left(3t\right)}{3}+\frac{\sin\left(5t\right)}{5}+\cdots\right]$

2. $\frac{8}{\pi}\left[\sin\left(t\right)+\frac{\sin\left(3t\right)}{3}+\frac{\sin\left(5t\right)}{5}+\ldots\right]$

3. (ii) $\frac{3}{2}+\frac{2}{\pi}\left[\sin\left(t\right)+\frac{\sin\left(3t\right)}{3}+\frac{\sin\left(5t\right)}{5}+\cdots\right]$

4. (ii) $\frac{80}{\pi}\left[\sin\left(t\right)+\frac{\sin\left(3t\right)}{3}+\frac{\sin\left(5t\right)}{5}+\ldots\right]$

6. (a) $\frac{8}{\pi}\left[\sin\left(t\right)+\frac{\sin\left(3t\right)}{3}+\frac{\sin\left(5t\right)}{5}+\ldots\right]$

(b) $\frac{20}{\pi}\left[\sin\left(t\right)+\frac{\sin\left(3t\right)}{3}+\frac{\sin\left(5t\right)}{5}+\ldots\right]$