Exercise **1(c)**

1. Construct a truth table for $(\neg P) \Rightarrow Q$.

2. Show that
(a) $(\neg P) \vee P$ is a tautology.
(b) $(\neg P) \wedge P$ is a contradiction.

3. Construct a truth table for

$$\left[(P \wedge (\neg Q)) \Rightarrow (R \wedge (\neg R))\right] \Rightarrow (P \Rightarrow Q)$$

What do you notice about your results?

4. Show that the following are tautologies:
(a) $\left[(P \Rightarrow Q) \wedge (P \Rightarrow R)\right] \Rightarrow \left[P \Rightarrow (Q \wedge R)\right]$
(b) $\left[(P \Rightarrow Q) \wedge (R \Rightarrow Q)\right] \Rightarrow \left[(P \vee R) \Rightarrow Q\right]$

5. State the contrapositive and converse of
(a) If x is even then x^2 is even
(b) If x^2 is odd then x is odd

6. Let x and y be real numbers. Let P be '$x = y$' and Q be '$x^2 = y^2$'. Write the following propositions in words and state which ones are true.
(i) $P \Rightarrow Q$
(ii) The converse of $P \Rightarrow Q$
(iii) The contrapositive of $P \Rightarrow Q$

7. Show that

$$\left[(P \Rightarrow Q) \wedge (\neg Q)\right] \Rightarrow (\neg P)$$

is a tautology.

8. Determine
(i) The converse of $P \Rightarrow (\neg Q)$
(ii) The contrapositive of $P \Rightarrow (\neg Q)$
(iii) Converse of the contrapositive of $(\neg P) \Rightarrow (\neg Q)$

9. Give an example of where $P \Rightarrow Q$ is true but the converse $Q \Rightarrow P$ is false.

10. State the contrapositive and converse of:
If $b^2 - 4ac \geq 0$ then the equation $ax^2 + bx + c = 0$ has real roots.

Also state whether the converse is true or false.

11. State the contrapositive and converse of:

$$\text{If } 1+a>0 \text{ then } (1+a)^n \geq 1+na$$