

Exercise 1(e)

1. Prove the following results:

- (a) $x^2 - 3x + 2 = 0 \Leftrightarrow x = 1$ or $x = 2$
 (b) $x^2 - 10x + 21 = 0 \Leftrightarrow x = 3$ or $x = 7$
 (c) $x^2 - 1 = 0 \Leftrightarrow x = 1$ or $x = -1$
 (d) $x^2 - (a+b)x + ab = 0 \Leftrightarrow x = a$ or $x = b$
 (e) $x^2 = y^2 \Leftrightarrow x = y$ or $x = -y$

2. Prove that $0 \mid a \Leftrightarrow a = 0$

3. Prove the following propositions:

- (a) n is even $\Leftrightarrow n^2$ is even
 (b) mn is odd \Leftrightarrow both m and n are odd
 (c) $m+n$ is odd \Leftrightarrow only m or only n is odd
 (d) mn is even \Leftrightarrow at least one of m or n is even

4. Let P and Q represent the following mathematical propositions. In each case decide whether $P \Rightarrow Q$ or $Q \Rightarrow P$ or $P \Leftrightarrow Q$. You do **not** have to prove any of these statements:

- (a) $P: a^2 > 0, \quad Q: a > 0$
 (b) $P: a \neq 0, \quad Q: a^2 > 0$
 (c) $P: x > 3, \quad Q: x > 4$
 (d) $P: x = 2$ or $x = -1, \quad Q: x^2 - x - 2 = 0$
 (e) $P: ax^2 + bx + c = 0$ has two real roots, $Q: b^2 - 4ac \geq 0$
 (f) $P: a \mid (b+c), \quad Q: a \mid b$ and $a \mid c$
 (g) $P: ac \mid bc, \quad Q: a \mid b$ where $c \neq 0$
 (h) $P: e^x = 1, \quad Q: x = 0$
 (i) $P: \ln(x) = 0, \quad Q: x = 1$
 (j) $P: 0 < a < b, \quad Q: a^n < b^n$ where n is a positive integer
 (k) $P: 0 < x < y, \quad Q: 0 < \frac{1}{y} < \frac{1}{x}$

(m) Consider the triangle below:

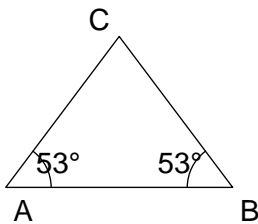


Fig 1

Let P : The triangle with equal angles $\sphericalangle A = \sphericalangle B$, Q : Lengths $AC = CB$

Brief Solutions to Exercise 1(e)

4. (a) $P \Leftrightarrow Q$ (b) $P \Leftrightarrow Q$ (c) $Q \Rightarrow P$
(d) $P \Leftrightarrow Q$ (e) $P \Leftrightarrow Q$ (f) $Q \Rightarrow P$
(g) $P \Leftrightarrow Q$ (h) $P \Leftrightarrow Q$ (i) $P \Leftrightarrow Q$
(j) $P \Rightarrow Q$ (k) $P \Leftrightarrow Q$ (m) $P \Leftrightarrow Q$