Exercise **3(c)**

1. Let $f:\mathbb{R}\to\mathbb{R}$ be given by

   (a) $f(x)=x$ (b) $f(x)=3x+1$ (c) $f(x)=x^3$

   (d) $f(x)=7x-2$ (e) $f(x)=1-x$

   Determine the inverse function $f^{-1}(x)$ in each case.

2. Let $f:\mathbb{R}^+\to\mathbb{R}^+$ be given by $f(x)=\sqrt{x}$. Show that this function is bijective and find the inverse function. [$f(x)=\sqrt{x}$ is the positive square root].

3. Let $f:\mathbb{R}/\{4\}\to\mathbb{R}/\{1\}$ be given by $f(x)=\dfrac{x+3}{x-4}$. Show that this function is bijective and find the inverse function.

4. Let $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x)=x^3+1$. Show that this function is bijective and find the inverse function.

5. Let A be the interval $[-1,\quad 1]$. Remember the interval $[-1,\quad 1]$ is the set of all real numbers between -1 to 1 inclusive. Let $f,\quad g$ and h be functions of $A\to A$ and defined by

   $$f(x)=x^2,\quad g(x)=\sin(x)\ \text{ and }\ h(x)=\cos(x)$$

   In each case decide whether they are bijective. If they are bijective then show that they are and state the inverse function.

   In the remaining questions the largest domain and codomain are subsets of $\mathbb{R}$.

6. Determine the largest domain and codomain so that the following functions are bijective:

   (a) $f(x)=\dfrac{1}{x}$ (b) $f(x)=\dfrac{3}{5-x}$ (c) $f(t)=\dfrac{2}{t+2}$

   (d) $f(x)=\dfrac{x-1}{2-x}$ (e) $f(x)=\dfrac{2x-1}{x+1}$

   In each case show that the function is indeed bijective and state the inverse function.

7. Determine the largest domain and codomain so that the following functions are bijective:

   (a) $f(x)=\dfrac{x+1}{3x+1}$ (b) $g(t)=\dfrac{1-t}{3t-1}$

   In each case show that the function is indeed bijective and state the inverse function. What do you notice about your result?

8. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=(x-3)^3+1$$

Prove that f is bijective and find a formula for f^{-1}.

9. Determine the largest domain and codomain so that the following functions are bijective:

(a) $f(x)=x^2-4x+3$ (b) $f(x)=9+8x-x^2$

(c) $f(x)=x^2+7x+1$ (d) $f(x)=2x^2+7x+1$

In each case show that the function is indeed bijective and state the inverse function.

Solutions to Exercise 3c

1. All functions are bijective.

(a) $f^{-1}(x)=x$ (b) $f^{-1}(x)=\dfrac{x-1}{3}$ (c) $f^{-1}(x)=\sqrt[3]{x}$

2. $f^{-1}(x)=x^2$

3. $f^{-1}(x)=\dfrac{4x+3}{x-1}$

4. $f^{-1}(x)=\sqrt[3]{x-1}$

5. f is neither injective nor surjective. The function g is injective but not surjective. h is neither injective nor surjective.

8. $f^{-1}(x)=\sqrt[3]{x-1}+3$