Exercise 7i

1. Determine the interval and radius of convergence of the following power series:

(a) $\sum_{n=0}^{\infty}\left(x^n\right)$ (b) $\sum_{n=1}^{\infty}\left(\frac{x^n}{2n}\right)$ (c) $\sum_{n=1}^{\infty}\left(\frac{x^n}{n^2}\right)$

(d) $\sum_{n=1}^{\infty}\left(\frac{nx^n}{2n+1}\right)$ (e) $\sum_{n=0}^{\infty}\left(\frac{x}{\sqrt{2}}\right)^n$ (f) $\sum_{n=0}^{\infty}\left(\frac{n^2}{2^n}\right)x^n$

2. Determine the interval of convergence of :

(a) $\sum_{n=0}^{\infty}\left(\frac{x^n}{n!}\right)$ (b) $\sum_{n=1}^{\infty}\left(\frac{(-1)^{n-1}x^n}{n}\right)$ (c) $\sum_{n=0}^{\infty}\left(\frac{(-1)^n x^{2n}}{(2n)!}\right)$

(d) $\sum_{n=0}^{\infty}\left(\frac{(-1)^n x^{2n+1}}{(2n+1)}\right)$ (e) $\sum_{n=0}^{\infty}\left(x^{2n}\right)$ (f) $\sum_{n=0}^{\infty}\left(x^{n^2}\right)$

3. Determine the interval of convergence of :

(a) $\sum_{n=1}^{\infty}\left(\frac{(x+1)^n}{n}\right)$ (b) $\sum_{n=1}^{\infty}\left(\frac{(x-2)^n}{n^2}\right)$ (c) $\sum_{n=0}^{\infty}\left(\frac{(-1)^n(x-1)^n}{2^n}\right)$

(d) $\sum_{n=0}^{\infty}\left(\frac{n}{2n+1}\left(\frac{x+3}{2}\right)^n\right)$

4. Show that $L=1$ for each of the series of part (III) of (7.30):

(a) $\sum\left(\frac{1}{n}\right)$ (b) $\sum\left(\frac{(-1)^{n+1}}{n^2}\right)$ (c) $\sum\left(\frac{(-1)^{n+1}}{n}\right)$

5. Find the interval of convergence of:

(a) $\sum_{n=0}^{\infty}\left(nx^{n-1}\right)$ (b) $\sum_{n=1}^{\infty}\left(\frac{(-1)^n x^n}{\sqrt{n}}\right)$ (c) $\sum_{n=0}^{\infty}\left(n^p x^n\right)$

(d) $\sum_{n=1}^{\infty}\left(\frac{x^n}{n^2\sqrt{n}}\right)$ (e) $\sum_{n=1}^{\infty}\left(\frac{(-3)^n x^n}{n\sqrt{n}}\right)$ (f) $\sum_{n=0}^{\infty}\left(x^n n^2 e^{-2n}\right)$

Solutions

1. (a) Interval of convergence is $-1<x<1$. Radius of convergence is $R=1$.
(b) Interval of convergence is $-1\le x<1$. Radius of convergence is $R=1$.
(c) Interval of convergence is $-1\le x\le 1$. Radius of convergence is $R=1$.
(d) Interval of convergence is $-1<x<1$. Radius of convergence is $R=1$.
(e) Interval of convergence is $-\sqrt{2}<x<\sqrt{2}$. Radius of convergence is $R=\sqrt{2}$.
(f) Interval of convergence is $-2<x<2$. Radius of convergence is $R=2$.
2. Interval of convergence is:
(a) $-\infty<x<+\infty$ (b) $-1<x\le 1$
(c) $-\infty<x<+\infty$ (d) $-1\le x\le 1$
(e) $-1<x<1$ (f) $-1<x<1$

3. Interval of convergence is
   (a) $-2 \le x < 0$ (b) $1 \le x \le 3$
   (c) $-1 < x < 3$ (d) $-5 < x < -1$
4. (a) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$ (b) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2 = 1$ (c) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$
5. Interval of convergence is
   (a) $-1 < x < 1$ (b) $-1 < x \le 1$ (c) $-1 < x < 1$ (d) $-1 \le x \le 1$
   (e) $-\frac{1}{3} \le x \le \frac{1}{3}$ (f) $-e^2 < x < e^2$