THE HATFIELD POLYTECHNIC

TABLE OF LAPLACE TRANSFORMS

Either $L[x(t)]$ is defined by $\int_0^\infty e^{-st}x(t)\,dt$ and is written $\bar{x}(s)$

or $L[f(t)]$ is defined by $\int_0^\infty e^{-st}f(t)\,dt$ and is written as $F(s)$.

No.	x(t) or f(t)	$\bar{x}(s)$ or F(s)
	$x(t) = L^{-1}[\bar{x}(s)]$ or $f(t) = L^{-1}[F(s)]$	$\bar{x}(s) = L[x(t)]$ or $F(s) = L[f(t)]$
1.	1	$\frac{1}{s}$
2.	e^{-at}	$\frac{1}{s+a}$
3.	$\sin bt$	$\frac{b}{s^2+b^2}$
4.	$\cos bt$	$\frac{s}{s^2+b^2}$
5.	t	$\frac{1}{s^2}$
6.	t^n (n a positive integer)	$\frac{n!}{s^{n+1}}$
7.	t^λ $(\lambda > -1)$	$\frac{\Gamma(\lambda+1)}{s^{\lambda+1}}$
8.	$\sinh bt$	$\frac{b}{s^2-b^2}$
9.	$\cosh bt$	$\frac{s}{s^2-b^2}$
10.	$t \sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
11.	$t \cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
12.	$\sin bt - bt\cos bt$	$\frac{2b^3}{(s^2+b^2)^2}$
13.	$e^{-at}t^n$ (n a positive integer)	$\frac{n!}{(s+a)^{n+1}}$
14.	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
15.	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
16.	$e^{-at}\sinh bt$	$\frac{b}{(s+a)^2-b^2}$
17.	$e^{-at}\cosh bt$	$\frac{s+a}{(s+a)^2-b^2}$

P.T.O.

No.	x(t) or f(t)	$\bar{x}(s)$ or F(s)
18.	$H(t)$	$\frac{1}{s}$
19.	$H(t-a)$	$\frac{e^{-as}}{s}$
20.	$\delta(t)$	1
21.	$\delta(t-a)$	e^{-as}
22.	$H(t-a)\,x(t-a)$	$e^{-as}\bar{x}(s)$
23.	$e^{-at}x(t)$	$\bar{x}(s+a)$
24.	$\frac{d}{dt}[x(t)]$	$s\bar{x}(s) - x(o)$
25.	$\frac{d^2}{dt^2}[x(t)]$	$s^2\bar{x}(s) - sx(o) - x'(o)$ or $s^2\bar{x}(s) - sx_o - x_1$
26.	$\frac{d^n}{dt^n}[x(t)]$	$s^n\bar{x}(s) - s^{n-1}x(o) - s^{n-2}x'(o) - \ldots\ldots - x^{(n-1)}(o)$ or $s^n\bar{x}(s) - s^{n-1}x_o - s^{n-2}x_1 - \ldots\ldots - x_{n-1}$
27.	$\int_o^t x(\tau)\,d\tau$	$\frac{1}{s}\bar{x}(s)$
28.	$-tx(t)$	$\frac{d}{ds}[\bar{x}(s)]$
29.	$(-t)^n x(t)$	$\frac{d^n}{ds^n}[\bar{x}(s)]$
30.	$\frac{1}{t}x(t)$	$\int_s^\infty \bar{x}(\sigma)\,d\sigma$
31.	$\int_o^t x(\tau)\,y(t-\tau)\,d\tau$	$\bar{x}(s)\,\bar{y}(s)$
32.	$x(t+T) = x(t)$	$\frac{1}{(1-e^{-Ts})}\int_o^T e^{-st}x(t)\,dt$

Note: $x_o \equiv x(o)$, $x_1 \equiv \left(\frac{dx}{dt}\right)_{t=o}$, ... $x_{n-1} \equiv \left(\frac{d^{n-1}x}{dt^{n-1}}\right)_{t=o}$, etc.

NOTES AND FORMULAE

(To be used together with the formulae in Castle's 5-figure tables)

TRIGONOMETRICAL AND HYPERBOLIC FUNCTIONS

$\sin^2 x + \cos^2 x = 1$; $\sec^2 x = 1 + \tan^2 x$; $\operatorname{cosec}^2 x = 1 + \cot^2 x$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x + y}{2} \sin \frac{y - x}{2}$$

$$\sin x = \frac{2t}{1 + t^2}; \quad \cos x = \frac{1 - t^2}{1 + t^2} \text{ where } t = \tan \frac{x}{2}$$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$; $\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$

$\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$

$\cosh^{-1} x = \pm \log(x + \sqrt{x^2 - 1})$

PARTIAL FRACTIONS

If degree of f(x) less than degree of denominator

$$\frac{f(x)}{(x + a)(x + b)\ldots\ldots} = \frac{A}{x + a} + \frac{B}{x + b} + \ldots\ldots\ldots\ldots$$

$$\frac{f(x)}{(ax^2 + bx + c)(dx + e)} = \frac{Ax + B}{ax^2 + bx + c} + \frac{C}{dx + e}$$

$$\frac{f(x)}{(ax + b)(cx + d)^2} = \frac{A}{ax + b} + \frac{B}{cx + d} + \frac{C}{(cx + d)^2}$$

DIFFERENTIATION

$y = \sinh x \qquad \frac{dy}{dx} = \cosh x$

$y = \cosh x \qquad \frac{dy}{dx} = \sinh x$

$y = \sinh^{-1} x \qquad \frac{dy}{dx} = \frac{1}{\sqrt{x^2 + 1}}$

$y = \cosh^{-1} x \qquad \frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$

MAXIMA AND MINIMA

If $y = f(x)$ then at $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} > 0$ there is a minimum

at $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} < 0$ there is a maximum

INTEGRATION

Reduction Formulae

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx$$

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \cos^{n-2} x \, dx$$

The integrals also apply with π and 2π at the upper limit.

Approximate Integration : Simpson's Rule

There must be an odd number of ordinates equally spaced

A = sum of first and last ordinates
B = sum of even ordinates
C = sum of odd ordinates
h = distance between ordinates

Area = h (A + 4B + 2C)/3

BINOMIAL THEOREM

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1.2} x^2 + \frac{n(n-1)(n-2)}{1.2.3} x^3 + \ldots\ldots$$

Case I: If n is a positive integer the series terminates

Case II: If n is not a positive integer the series is infinite and converges if $-1 < x < 1$.

TAYLOR'S THEOREM

$$f(a + h) = f(a) + hf'(a) + \frac{h^2}{1.2} f''(a) + \frac{h^3}{1.2.3} f'''(a) + \ldots\ldots$$

Newton's Approximation to the root of an equation:

If $x = h$ is an approximate solution of $f(x) = 0$ then usually

$h - \frac{f(h)}{f'(h)}$ is a better approximation.