Exercise **4(d)**

Let a, b, c *and* d *be real numbers.*

1. Prove that if $a<b$ then $a-c<b-c$.
2. Prove that if $a<b$ and $b<c$ then $a<c$.
3. Prove that if $a<b$ and $c\le d$ then $a+c<b+d$.
4. Let $x\in\mathbb{R}$ prove that

   (a) if $x>0$ then $\frac{1}{x}>0$

   (b) if $x<0$ then $\frac{1}{x}<0$
5. Let $a\in\mathbb{R}$ and $b\in\mathbb{R}$ show that

   (a) if $0<a<b$ then $\frac{1}{a}>\frac{1}{b}$.

   (b) if $b<a<0$ then $\frac{1}{b}>\frac{1}{a}$.
6. Let $x\in\mathbb{R}$, prove that $x^2\ge 0$.
7. Let $x\in\mathbb{R}$, show that

   (a) if $a<b$ then $ax^2\le bx^2$.

   (b) if $a<b$ then $-ax^2\ge -bx^2$.
8. Let $x\in\mathbb{R}$, show that $x^2-2x+1\ge 0$.
9. Let $x\in\mathbb{R}$, show that $x^2-5x+9\ge\frac{11}{4}$. [Hint: Use Completing the Square].
10. Prove that $(a+b)^2\ge 4ab$.
11. Let $x\in\mathbb{R}$ and $y\in\mathbb{R}$, prove that

$$\left[\frac{1}{2}(x+y)\right]^2\le\frac{1}{2}\left(x^2+y^2\right)$$

12. Let $n\in\mathbb{N}$,disprove that $(n+1)^2\ge 2n^2$.

(Challenge: Prove that $(n+1)^2\le 2n^2$ *for* $n\ge 3$*).*

13. Let $x\in\mathbb{R}$, prove that if $x>1$ then $x^n\ge x$ for all $n\in\mathbb{N}$.
14. Let $x\in\mathbb{R}$, prove that if $0<x<1$ then $x^n\le x$ for all $n\in\mathbb{N}$.
15. Let $x\in\mathbb{R}$ and $£>0$ be a real number. Prove that if for every $£>0$ we have

$3\le x<3+£$ then $x=3$.

16. Let $x \in \mathbb{R}$ and $£ > 0$ be a real number. Prove that if for every $£ > 0$ we have

$a \leq x < a + £$ then $x = a$.

17. Let $x \in \mathbb{R}$ and $£ > 0$ be a real number. Prove that if $x > 0$ then there is an$£ > 0$ such that $x \geq £$.

18. Let $x \in \mathbb{R}$ and $y \in \mathbb{R}$ prove that if $0 < x < 1$ and $0 < y < 1$ then

$$\frac{x+y}{1+xy} < 1$$