2nd Order Differential Eqns

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(1)

Homogenous Eqns

Section 11.4B

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$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

where $a \neq 0$.

$$\text{---} \quad \text{---} \quad \text{---------} = 0.$$

↑ homogeneous

Solve

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0.$$

$y = y(x)$

Try $y = e^{mx}$

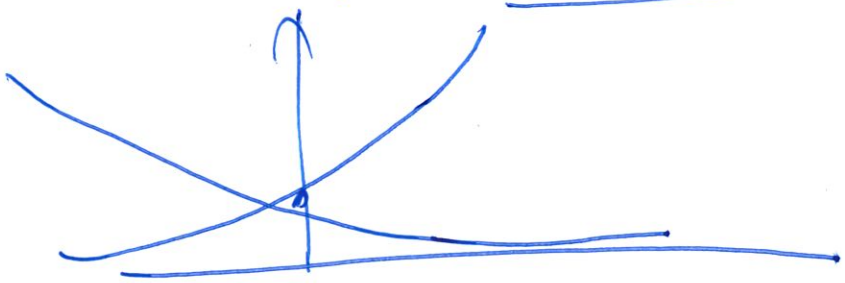
$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

$$am^2e^{mx} + bme^{mx} + ce^{mx} = 0.$$

$$e^{mx}\left[am^2 + bm + c\right] = 0.$$

$e^{mx} \neq 0$ $\quad am^2 + bm + c = 0$



$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$am^2 + bm + c = 0.$$

(i) Real & different m_1 & m_2

(ii) Real & Equal $m_1 = m_2 = m$

(iii) Complex $m = \alpha \pm j\beta$.

$$y_1(x) = e^{m_1 x} \qquad y_2(x) = e^{m_2 x}$$

$$y = Ae^{m_1 x} + Be^{m_2 x}$$

$$y = (A + Bx)e^{mx}$$

$$y = e^{\alpha x}\left[A\cos(\beta x) + B\sin(\beta x)\right]$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Solve $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 3y = 0$

Soln:

$$m^2 + 2m - 3 = 0$$

$$(m+3)(m-1) = 0$$

$$m_1 = -3, \quad m_2 = 1$$

$$y = Ae^{-3x} + Be^{x}$$ ← general soln

Solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$$

Soln:

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = -1$$

$$y = (A + Bx)e^{-x}.$$

Find the general soln of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0.$$

Soln:

$$m^2 + m + 1 = 0.$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm j\sqrt{3}}{2}$$

$$m = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\alpha \qquad\qquad \beta.$$

$$y = e^{\alpha x}\left[A\cos(\beta x) + B\sin(\beta x)\right]$$

$$= e^{-\frac{1}{2}x}\left[A\cos\left(\frac{\sqrt{3}}{2}x\right) + B\sin\left(\frac{\sqrt{3}}{2}x\right)\right]$$

Solve

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + \cos(x) = 0.$$

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = -\cos(x)$$