Proof: WLOG let A be an upper Δ matrix:

Pages 507

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \text{———} & a_{2} \\ \vdots & 0 & \ddots & \\ 0 & \vdots & & a_{nn} \end{pmatrix}.$$

R.t.p. $\boxed{\lambda_1 = a_{11}, \quad \lambda_2 = a_{22} \cdots \lambda_n = a_{nn}.}$

$$\det(A - \lambda I) = \det \begin{pmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ 0 & a_{22}-\lambda & \text{——} & a_{2n} \\ 0 & 0 & \ddots & \\ \vdots & & & a_{nn}-\lambda \end{pmatrix}$$

$$= (a_{11}-\lambda)(a_{22}-\lambda)\cdots(a_{nn}-\lambda) = 0$$

Hence $\lambda_1 = a_{11}$, $\lambda_2 = a_{22}$, $\cdots$ $\lambda_n = a_{nn}$.

Page 504

Proof of (a): Let $\underline{y}$ be an e. vector belonging to λ of A.

$$A\underline{y} = \lambda \underline{y}$$

R.t.p. $\boxed{A^m \underline{y} = \lambda^m \underline{y}}$

Proof by induction.

Step 1: Check for $m=1$:

$$A\underline{y} = \lambda \underline{y} \quad \checkmark$$

Step 2: Assume the result is true for $m=k$:

$$A^k \underline{y} = \lambda^k \underline{y} \qquad (*)$$

Step 3: We need to prove that

$$\boxed{A^{k+1} \underline{y} = \lambda^{k+1} \underline{y}}$$

Consider the LHS:

$$\begin{aligned} A^{k+1}\underline{y} &= A\left(A^k \underline{y}\right) \\ &= A\left(\lambda^k \underline{y}\right) \\ &= \lambda^k\left(A\underline{y}\right) \\ &= \lambda^k\left(\lambda \underline{y}\right) \\ &= \lambda^{k+1}\underline{y} \end{aligned}$$

Δ

The eigenvalues of A are $1, 2, 3$.

—————— A^7 are

$1^7, 2^7, 3^7$.