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$$\cancel{D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad D^{-1} = \begin{pmatrix} 1 & & 0 \\ & 1/2 & \\ 0 & & 1/3 \end{pmatrix}$$

$$D^{100} = \begin{pmatrix} 1 & & 0 \\ & 2^{100} & \\ 0 & & 3^{100} \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = D.$$

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}.$$

Proof: We are given $A$ & $B$ are similar.

By defn we have

$$P^{-1}AP = B.$$

R.T.P. $\boxed{\det(B-\lambda I) = \det(A-\lambda I)}$

Consider the LHS.

$$\begin{aligned}
\underline{\det(B-\lambda I)} &= \det\left(P^{-1}AP - \lambda I\right) \\
&= \det\left(P^{-1}AP - \lambda P^{-1}P\right) \\
&= \det\left[P^{-1}(A-\lambda I)P\right] \\
&= \det(P^{-1})\det(A-\lambda I)\det(P) \\
&= \underbrace{\det(P^{-1})\det(P)}_{=1}\det(A-\lambda I) \\
&= \det(A-\lambda I)
\end{aligned}$$

□