

Taylor Series of Functions Page 365.

Find the TS for $\ln(x)$ about $x=2$.

Soln, Let $a=2$, $f(x) = \ln(x)$.

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x} = x^{-1}$$

$$f''(x) = -x^{-2} = -\frac{1}{x^2}$$

$$f'''(x) = 2x^{-3} = \frac{2}{x^3}$$

$$f^{(4)}(x) = -6x^{-4} = -\frac{6}{x^4}$$

$$f(2) = \ln(2)$$

$$f'(2) = \frac{1}{2}$$

$$f''(2) = -\frac{1}{2^2} = -\frac{1}{4}$$

$$f'''(2) = \frac{2}{2^3} = \frac{1}{4}$$

$$f^{(4)}(2) = \frac{-6}{2^4} = -\frac{6}{16} \\ = -\frac{3}{8}$$

$$\ln(x) = f(a) + f'(a)(x-a)$$

$$+ \frac{f''(a)}{2!} (x-a)^2 + \dots$$

$$\ln(x) = \ln(2) + \frac{1}{2}(x-2) + \frac{-1/4}{2!}(x-2)^2$$

$$+ \frac{1/4}{3!}(x-2)^3$$

$$+ \frac{-3/8}{4!}(x-2)^4 + \dots$$

$$= \ln(2) + \frac{x-2}{2} - \frac{(x-2)^2}{8} + \frac{(x-2)^3}{24}$$

$$- \frac{3(x-2)^4}{64}$$