Appendix A

Algebraic Properties of Real Numbers

Consider two real numbers x and y. Addition of x and y is denoted by $x+y$ and multiplication by xy.

Addition of real numbers x, y and z has the following properties:

A1. Commutative – The order of addition does **not** matter:

$$x+y=y+x.$$

A2. Associative – The way we add in groups does **not** matter:

$$\left(x+y\right)+z=x+\left(y+z\right).$$

A3. Neutral Element – There is a real number 0 called the zero element such that for every real number x we have

$$x+0=x.$$

A4. Inverse Element – For every real number x there is a real number $-x$, pronounced minus x, such that

$$x+\left(-x\right)=0.$$

Multiplication of real numbers x, y and z has the following properties:

M1. Commutative – The order of multiplication does **not** matter:

$$xy=yx.$$

M2. Associative – The order of the group of multiplication does **not** matter:

$$\left(xy\right)z=x\left(yz\right).$$

M3. Neutral Element – There is a real number 1 called the unit (or identity) element such that for every real number x we have

$$x\left(1\right)=x.$$

M4. Inverse Element – For every real number x apart from 0, $x\neq 0$, we have an inverse element denoted by $\frac{1}{x}$ such that

$$x\times\left(\frac{1}{x}\right)=1 \text{ provided } x\neq 0.$$

Connecting the two operations, addition and multiplication, we have:

D1. Distributive – Multiplication has priority over addition. That is for any real numbers x, y and z we have

$$x\left(y+z\right)=xy+xz \quad\text{and}\quad \left(x+y\right)z=xz+yz.$$