Miscellaneous Exercise 7 question 21. Page 565.

(b) $$\underline{v}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \underline{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \underline{v}_3 = \begin{pmatrix} 5 \\ 4 \\ 6 \end{pmatrix}$$

Apply G-S to find an orthonormal set in $\mathbb{R}^3$.

$$\underline{w}_1 = v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

$$w_2 = v_2 - \frac{v_2 \cdot w_1}{\|w_1\|^2} \underline{w}_1 \qquad (*)$$

$$v_2 \cdot w_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 1$$

$$\|w_1\|^2 = 0^2 + 1^2 + 1^2 = \underline{2}$$

Substituting into $(*)$ yields

$$w_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - 0 \\ 1 - \frac{1}{2} \\ 0 - \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1/2 \\ -1/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Let $\underline{w_2^*} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}.$

We have

$$w_3 = v_3 - \frac{v_3 \cdot w_1}{\|w_1\|^2} w_1 - \frac{v_3 \cdot w_2^*}{\|w_2^*\|^2} w_2^* \qquad (+)$$

$$v_3 \cdot w_1 = \begin{pmatrix} 5 \\ 4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0 + 4 + 6 = \underline{10}$$

$$v_3 \cdot w_2^* = \begin{pmatrix} 5 \\ \cdots \end{pmatrix} \cdot \begin{pmatrix} 2 \\ \cdots \end{pmatrix} = \cdots$$

$$v_3 \cdot w_2^* = \begin{pmatrix}5\\4\\6\end{pmatrix} \cdot \begin{pmatrix}2\\1\\-1\end{pmatrix} = 10 + 4 - 6 = 8$$

$$\|w_2^*\|^2 = \left\|\begin{pmatrix}2\\1\\-1\end{pmatrix}\right\|^2 = 2^2 + 1^2 + (-1)^2 = 6$$

Substituting into (+) gives.

$$w_3 = \begin{pmatrix}5\\4\\6\end{pmatrix} - \frac{10}{2}\begin{pmatrix}0\\1\\1\end{pmatrix} - \frac{8}{6}\begin{pmatrix}2\\1\\-1\end{pmatrix}$$

$$= \begin{pmatrix}5\\4\\6\end{pmatrix} - 5\begin{pmatrix}0\\1\\1\end{pmatrix} - \frac{4}{3}\begin{pmatrix}2\\1\\-1\end{pmatrix}$$

$$= \begin{pmatrix}5 - 0 - 8/3\\4 - 5 - 4/3\\6 - 5 + 4/3\end{pmatrix}$$

$$= \begin{pmatrix}7/3\\-7/3\\7/3\end{pmatrix} = \frac{7}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$w_3' = \begin{pmatrix}1\\-1\\1\end{pmatrix}.$$

$$w_1 = \begin{pmatrix}0\\1\\1\end{pmatrix}, \quad w_2^* = \begin{pmatrix}2\\1\\-1\end{pmatrix}, \quad w_3' = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$\hat{w}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$\hat{w}_2^* = \frac{1}{\sqrt{6}}\begin{pmatrix}2\\1\\-1\end{pmatrix}$$

$$\hat{w}_3' = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix}.$$

$- s \qquad \sqrt{3}\,|\,{}^{1}_{1}\,|.$

c. Let $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$

(i) Find the e.values of A.

(ii) e.vectors for each e.value.

$\lambda_{1,2} = -1$ and $\lambda_3 = 8$.

Let $\underline{u}$ be the e.vector belonging to $\lambda_{1,2} = -1$.

$$(A + I)\underline{u} = \begin{pmatrix} 3+1 & 2 & 4 \\ 2 & 0+1 & 2 \\ 4 & 2 & 3+1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$4x + 2y + 4z = 0$$

$$2x + y + 2z = 0$$

$\underline{u}_1 = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}$; $u_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ for $\lambda_4 = -1$.

$\lambda_3 = 8$

$$\begin{pmatrix} 3-8 & 2 & 4 \\ 2 & 0-8 & 2 \\ 4 & 2 & \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$4x + 2y - 4z = 0$$

$$\underline{v} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}.$$