Example 12

construct the truth table for $p \vee (\neg p)$.

Soln:

p	$\neg p$	$p \vee (\neg p)$
T	F	T
F	T	T

$p \vee (\neg p)$ is always true & denote by

$$p \vee (\neg p) \equiv T$$

a tautology is always true

$p: x^2 - 9 = 0$ then

$p \vee (\neg p)$: $x^2 - 9 = 0$ or $x^2 - 9 \neq 0$.

Example 13

Construct the truth table for $(P \Rightarrow Q) \wedge (Q \Rightarrow R)$

Soln:

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
F	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

Example 15

Construct the truth table for $Q \Rightarrow P$.

P	Q	$Q \Rightarrow P$	$P \Rightarrow Q$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	T	T

$$(P \Rightarrow Q) \not\equiv (Q \Rightarrow P)$$

not equivalent.

Example 16 If n is even then n^2 is even.

n is even $\Rightarrow$ n^2 is even.

assume this

deduce that

Proof Assume n is even. So we have

$$n = 2m$$

$$n^2 = 4m^2 = 2(2m^2)$$

$$n^2 = 2 \times \text{integer}$$

$\Rightarrow$ n^2 is even.

QED

Example 17

I.4 The sum of two odd numbers is even.

Proof Consider m and n are odd.

$$m = 2k+1$$
$$n = 2\ell+1$$

$$\begin{aligned} m+n &= 2k+1 + 2\ell+1 \\ &= 2(k+\ell) + 2 \\ &= 2[k+\ell+1] \\ &= 2 \times (\text{integer}) \Rightarrow m+n \text{ is even.} \end{aligned}$$

$\square$

Example 18

If $a \mid b$ and $b \mid c$ then $a \mid c$.

$\underbrace{a \mid b \text{ and } b \mid c}_{P} \Rightarrow \underbrace{a \mid c}_{Q}$

Proof: Assume $a \mid b$

$$\underline{ax = b}$$

$b \mid c$

$$by = c \qquad (*)$$

Substituting $b = ax$ into $(*)$ gives

$$b(ax) = c$$

$$\underline{a(bx) = c}$$

$a \times \text{integer} = c$
$\Rightarrow a \mid c.$

By (I.5) we have $a \mid c$.

$\square$